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1995 J. Phys.: Condens. Matter 7 6833

(http://iopscience.iop.org/0953-8984/7/34/008)

J. Phys.: Condens. Matter 7 (1995) 6833–6845. Printed in the UK

The torque exerted on a narrow slab of ^{3}He-A in a perpendicular magnetic field

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Received 29 April 1994, in final form 2 March 1995

Abstract. The hydrodynamic equations for a narrow slab when the state of ^{3}He-A is deformed slightly by a magnetic field are solved. Then the torque exerted on the torsional oscillator in which the slab is contained is calculated, and the changes in the resonant frequency as a function of the magnetic field are determined.

1. Introduction

The unusual intrinsic anisotropy of superfluid ^{3}He-A is characterized by two unit vectors $\hat{l}$, associated with the orbital angular momentum, and $\hat{d}$, associated with the spin angular momentum of the triplet p-wave Cooper pairs. The magnetic deformation of the textures in a narrow slab has been considered by Fetter [1], who obtained the phase diagram for superfluid ^{3}He-A confined in a narrow slab, subject to a perpendicular magnetic field, by minimizing the total free energy and using the variational approach.

For a magnetic field $\boldsymbol{H} = H\hat{Z}$ perpendicular to the face of the slab which is smaller than H_D, the dipole magnetic field, i.e. $h = H/H_D \ll 1$, the dipole locking aligns $\hat{d}$ parallel to $\hat{l}\|\hat{Z}$, $\theta = \phi = 0$ whereas, for $h \gg 1$, the magnetic force aligns $\hat{d}$ perpendicular to $\hat{l}$, $\theta = 0$ and $\phi = \pi/2$. In both these regimes the resulting texture is uniform across the slab. For intermediate fields, however, the texture undergoes a transition to a deformed state, analogous to the Freedericksz transition observed in nematic liquid crystals [1, 2]. The boundary curve characterizing the onset of deformation has been calculated exactly [3]. There are two critical fields: h_{c_1}, for the onset of deformation and h_{c_2}, a second critical field below which the texture is non-uniform, i.e. for $h_{c_1} < h < h_{c_2}$ we have a deformed state.

The textures in the slab has also been considered by Hook and co-workers [4]; they obtained an exact numerical solution of the Euler equations for $\theta(Z)$ and $\phi(Z)$. Extrapolating the exact numerical solutions of θ and ϕ for a very thin slab indicates that the agreement between these two approaches are good.

The instability of the uniform orbital texture of a slab of ^{3}He-A in a perpendicular magnetic field has been observed by Hook and co-workers [2, 4]. The slab width d in their experiments was 105 μm which is greater than the critical width d_c of the slab. The upper curve for h_{c_2} tends to infinity at this critical width. For observation of both threshold magnetic fields h_{c_1} and h_{c_2} in the textural phase diagram of superfluid ^{3}He-A confined in a slab, one hence needs to use a thin slab in which the width is smaller than d_c.

In this paper we consider that part of the textural phase diagram in which $d < d_c$, and the uniform orbital texture deforms slightly. First we write briefly the Euler equations for $\theta(Z)$ and $\phi(Z)$ and their variational solutions. We next calculate the hydrodynamic equations and obtain the components of the normal and superfluid velocities. Finally we compute the torque exerted on the slab and give some concluding remarks.

2. Variational solution for a narrow slab

Consider a slab of ^{3}He-A inside a torsional oscillator of width d, placed in a perpendicular magnetic field H. We shall assume that the orbital texture is not affected by the flow associated with the motion of the torsional oscillator. This can be arranged in an experiment by keeping the amplitude of oscillation sufficiently small [4]. Furthermore, we assume that the uniform texture which is deformed by a perpendicular magnetic field is a planar texture, which is the case at small oscillation amplitudes. Therefore, in the present slab geometry, the vectors $\hat{l}$ and $\hat{d}$ are coplanar, and in the Z–y plane, specified by the pair of angular variables θ and ϕ, $\hat{l}\cdot\hat{Z} = \cos\theta$ and $\hat{d}\cdot\hat{Z} = \cos\phi$. The equilibrium orientation of the $\hat{l}$ and $\hat{d}$ vectors is determined by minimizing the free energy associated with the free-energy density [5]:

$$f = \frac{L_D^2}{2}\left\{(K_5\sin^2\theta + K_b\cos^2\theta)\left(\frac{\mathrm{d}\theta}{\mathrm{d}z}\right)^2 + \left[\left(\frac{\rho_{sp\perp}}{\rho_{sp\parallel}}\right)\sin^2\theta + \cos^2\theta\right]\left(\frac{\mathrm{d}\phi}{\mathrm{d}z}\right)^2\right\} + \tfrac{1}{2}\sin^2(\theta-\phi) + \tfrac{1}{2}h^2\cos^2\phi. \qquad (1)$$

f is dimensionless, measured in units of $\rho_{sp\parallel}L_D^2$. Here, the various bending coefficients are dimensionless dipole-unlocked values, measured in units of $\rho_{sp\parallel}$. The first two terms in equation (1) are the bending energies, the third is the dipole contribution, and the last is the magnetic term. $L_D = (\rho_{sp\parallel}/\lambda_D)^{1/2}$ is the dipole-unlocking length (about 10 μm). The problem is to minimize the total free energy $\langle f\rangle = \int \mathrm{d}z f$, subject to the usual hydrodynamic boundary conditions.

As we mentioned previously, the variational approach [1] for minimizing equation (1) for the case we are considering is in agreement with exact numerical calculation [4]. From the symmetry considerations of $\theta(z)$ about the centre of the slab and the boundary conditions on $\theta(z)$ and $\phi(z)$ it is convenient to use a sinusoidal form for

$$\theta(z) = A_\theta\cos\left(\frac{\pi}{d}z\right) \qquad (2)$$

and

$$\phi(z) = B_\phi \qquad (3)$$

where A_θ and B_ϕ are determined variationally. This calculation has already been done by Fetter [1] and we quote his results:

$$\langle f\rangle = (\pi^2A_\theta^2/8D^2)\{K_b + K_s + (K_b - K_s)[J_0(2A_\theta) + J_2(2A_\theta)]\} + \tfrac{1}{4}(h^2+1) - [\tfrac{1}{16}(h^2 - J_0(2A_\theta))^2 + \pi^{-2}F^2(A_\theta)]^{1/2} \qquad (4)$$

where $F(A) = \sum_{K=0}^{\infty}(2K+1)^{-1}J_{2K+1}(2A_\theta)$, J_n denotes the usual Bessel functions and $D = d/LD$. Equation (4) should be minimized with respect to the amplitude A_θ for a fixed value of applied magnetic field h. By using the values of H_D, L_D, K_s and K_b from table 1 of [1] we obtain the numerical values of A_θ at different magnetic fields for a slab of width 25 μm. A_θ^2 is plotted versus h in figure 1 for two different values of reduced temperature. As is seen from the figure the maximum value of A_θ decreases with decreasing temperature.

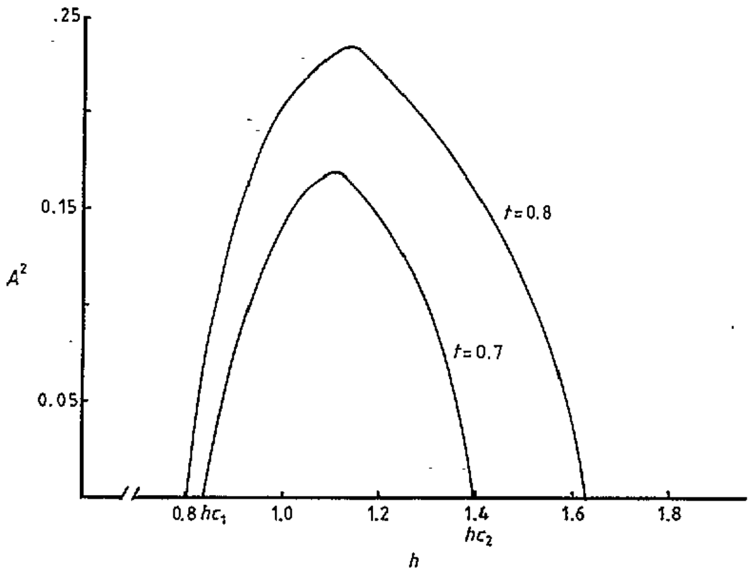

Figure 1. The calculated values of A_θ^2 versus reduced magnetic field h.

3. The hydrodynamic equations of a narrow slab of ^{3}He-A in torsional oscillator

We now proceed to solve the hydrodynamic equations of a narrow slab of ^{3}He-A in a torsional oscillator with small oscillation amplitudes for obtaining the superfluid and normal fluid velocities $\boldsymbol{V}^s$ and $\boldsymbol{V}^n$. We assume that the total density of the fluid remains constant during the oscillator motion. Then the conservation of mass requires that

$$\nabla \cdot g = -\frac{\partial \rho}{\partial t} = 0 \tag{5}$$

where the current density is [6]

$$g = \rho^n \cdot V^n + \rho^s \cdot V^s + \mathbf{C} \cdot \nabla \times \hat{l}. \tag{6}$$

The components of the tensor $\mathbf{C}$ are written as $C_{ij} = C\delta_{ij} + C_0\hat{l}_i \cdot \hat{l}_j$. For the configuration that we are considering, one can easily show that the contribution of the third term on the right-hand side of equation (6) to the equation (5) is zero. Equations (5) and (6), hence, give

$$\nabla \cdot (\rho^n \cdot V^n + \rho^s \cdot V^s) = 0. \tag{7}$$

The equations of motion of g and V^s linearized to first order in the velocities can be written as

$$\frac{\partial g_j}{\partial t} + \nabla_i \Pi_{ij} + \nabla_j P = \mathbf{0} \tag{8}$$

and

$$\frac{\partial V^s}{\partial t} + \nabla(\mu + \zeta_3 \nabla \cdot (g - \rho V^n)) = \mathbf{0} \tag{9}$$

where we have ignored many dissipative and reactive coefficients [7] in writing the equations in this form. Microscopic calculations suggest [8] that forces resulting from these are negligible in comparison with those from the stress tensor Π_{ij}. ζ_3 is the second viscosity term and μ is the chemical potential. The stress tensor can be written as

$$\Pi_{ij} = -\eta_{ijkl}(\nabla_k \cdot V_l^n + \nabla_l \cdot V_k^n - \tfrac{2}{3}\delta_{lk}\nabla \cdot V^n) \tag{10}$$

where the shear viscosity coefficients η_{ijkl} may be written as [9]

$$\tfrac{1}{2}\eta_{ijkl} = A(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) + B(\delta_{ik}l_j l_l + \delta_{jk}l_i l_l + \delta_{il}l_j l_k + \delta_{jl}l_i l_k) + C\delta_{ij}\delta_{kl} + Dl_i l_j l_k l_l + E(\delta_{ij}l_k l_l + \delta_{kl}l_i l_j). \tag{11}$$

Alternative choices of the five coefficients are also defined by Wolfle [10] and Hook and co-workers [4].

Following the argument of Hook and co-workers [4] about the smallness of the fountain effect term $S\,\nabla T$, which was not included in equation (9) we may relate the pressure and chemical potential as

$$\nabla P = \rho\,\nabla\mu. \tag{12}$$

Equation (12) allows us to eliminate P and μ from the problem and to replace equations (8) and (9) by

$$\frac{\partial}{\partial t}(g_j - \rho V_j^s) + \nabla_i \Pi_{ji} + \rho^2\zeta_3\nabla_j(\nabla \cdot V_n) = 0 \tag{13}$$

The coefficient ζ_3 diverges at T_c and effects due to this term might be visible near T_c—a region that we do not consider in this paper. The third term in (13), hence, is negligible in comparison with those from the shear viscosity tensor Π_{ji}.

Another useful relation which connects the different components of the superfluid velocity to each other is the Mermin–Ho [11] relation

$$\nabla_j \cdot V_i^s - \nabla_i \cdot V_j^s = \frac{\hbar}{2m}\hat{l} \cdot (\nabla_j \cdot \hat{l} \times \nabla_i \cdot \hat{l}). \tag{14}$$

For the case under consideration the term on the right-hand side of equation (14) is zero. The aim of our calculation is to compute the torque exerted by the fluid on the oscillator, and we, therefore, need to solve equations (7), (13) and (14) with suitable boundary conditions. To solve the hydrodynamics of the planar texture, we assume velocity fields of the form

$$V^n = (V^n(y,z)\hat{x} + V_y^n(x,z)\hat{y} + V_z^n(x,z)\hat{Z})\exp(\mathrm{i}\omega t) \tag{15}$$

and

$$V^s = (V_x^s(z,y)\hat{x} + V_y^s(x,z)\hat{y} + V_z^s(x,z)\hat{z})\exp(\mathrm{i}\omega t) \tag{16}$$

where we use Cartesian coordinates and ω is the frequency of oscillation of the slab. By using equations (10), (11), (15) and (16) in (13) we obtain the following equations for the planar texture:

$$\mathrm{i}\omega(\rho_\perp^s V_x^s + \rho_\perp^n V_x^n - \rho V_x^s) + E\sin(2\theta)\frac{\partial^2 V_y^n}{\partial z\partial x} + B\sin(2\theta)\frac{\partial^2 V_x^n}{\partial z\partial y} + B\frac{\partial}{\partial z}\left[\sin(2\theta)\left(\frac{\partial V_y^n}{\partial x} + \frac{\partial V_x^n}{\partial y}\right)\right] + 2\frac{\partial}{\partial z}\left((A + B\cos^2\theta)\frac{\partial V_x^n}{\partial z}\right) + 2\frac{\partial}{\partial z}\left((A + B\cos^2\theta)\frac{\partial V_z^n}{\partial x}\right) + 2\frac{\partial}{\partial x}\left((C + E\cos^2\theta)\frac{\partial V_z^n}{\partial z}\right) = 0 \tag{17}$$

$$\mathrm{i}\omega\{\tfrac{1}{2}(\rho_\parallel^s - \rho_\perp^s)\sin(2\theta)V_z^s + [\rho_\perp^s + (\rho_\parallel^s - \rho_\perp^s)\sin^2\theta]V_y^s + \tfrac{1}{2}(\rho_\parallel^n - \rho_\perp^n)\sin(2\theta)V_z^n + [\rho_\perp^n + (\rho_\parallel^n - \rho_\perp^n)\sin^2\theta]V_y^n - \rho V_y^s\} + 2(C + D + E)\frac{\partial^2 V_y^n}{\partial z^2}$$

$$+2\frac{\partial}{\partial z}\left(B\sin(2\theta)+\frac{D}{2}\sin(2\theta)\cos^2\theta+\frac{E}{2}\sin(2\theta)\right)\frac{\partial V_z^n}{\partial z}=0 \tag{18}$$

and

$$\begin{aligned}
&\mathrm{i}\omega\{[\rho_\perp^s+(\rho_\parallel^s-\rho_\perp^s)\cos^2\theta]V_z^s+\tfrac{1}{2}(\rho_\parallel^s-\rho_\perp^s)\sin(2\theta)V_y^s+\tfrac{1}{2}(\rho_\parallel^n-\rho_\perp^n)\sin(2\theta)V_y^n\\
&\quad+[\rho_\perp^n+(\rho_\parallel^n-\rho_\perp^n)\cos^2\theta]V_z^n-\rho V_z^s\}\\
&\quad+2\frac{\partial}{\partial z}\left(B\sin(2\theta)+\frac{D}{2}\sin(2\theta)\cos^2\theta+\frac{E}{2}\sin(2\theta)\right)\frac{\partial v_y^n}{\partial z}\\
&\quad+2\frac{\partial}{\partial z}(2A+4B\cos^2\theta+C+D\cos^4\theta+2E\cos^2\theta)\frac{\partial V_z^n}{\partial z}=0
\end{aligned} \tag{19}$$

and equation (7) becomes

$$\begin{aligned}
&(\rho_\parallel^s-\rho_\perp^s)\sin\theta\cos\theta V_y^s+[\rho_\perp^s+(\rho_\parallel^s-\rho_\perp^s)\cos^2\theta]V_z^s+(\rho_\parallel^n-\rho_\perp^n)\sin\theta\cos\theta V_y^n\\
&\quad+[\rho_\perp^n+(\rho_\parallel^n-\rho_\perp^n)\cos^2\theta]V_z^n=0.
\end{aligned} \tag{20}$$

As we mentioned previously, we are considering the state of ^{3}He-A in a torsional oscillator which is deformed slightly. On the basis of this assumption and the symmetry of the problem it is possible to use the following approximations for the velocity field components:

$$\begin{aligned}
&V_x^n(y,z)\simeq ya_0^n(z)+A_\theta a_1^n(z)+A_\theta^2 ya_2^n(z)\\
&V_y^n(x,z)\simeq xb_0^n(z)+A_\theta xb_1^n(z)+A_\theta^2 xb_2^n(z)\\
&V_z^n(x,y)\simeq xC_0^n(z)+A_\theta xC_1^n(z)+A_\theta^2 xC_2^n(z)\\
&V_x^s(z)\simeq a_0^s(z)+A_\theta a_1^s(z)+A_\theta^2 a_2^s(z)+A_H y\\
&V_z^s(x,z)\simeq xC_0^s(z)+A_\theta xC_1^s(z)+A_\theta^2 xC_2^s(z)\\
&V_y^s(x,z)\simeq A_H x.
\end{aligned} \tag{21}$$

Since we are considering the planar texture which is deformed slightly from the uniform texture, we may use the following boundary conditions [4]: $\boldsymbol{V}^n=r\omega\hat{\phi}$, $g_z=\boldsymbol{0}$ at $z=\pm d/2$ and $\int_{-d/2}^{d/2}g_r\,\mathrm{d}z=\boldsymbol{0}$. In obtaining the last equation in (21) we used equation (14) and the boundary conditions (see appendix).

By substituting equations (21) into equations (17)–(20) and ignoring terms which are proportional to A_θ^3 or higher order we get the following equation for each component of the velocities (see appendix):

$$\frac{\partial^2}{\partial z^2}V_{ij}^{n,s}+K_j^2V_{ij}^{n,s}=f_{ij}^{n,s}(z) \tag{22}$$

where i stands for the x, y and z components of the velocities and j for 0, 1 and 2 (the zeroth, first and second orders of the velocity components). The inhomogeneous second-order differential equation (22) has been solved by the method of variations in constants [12]. Since these solutions are straightforward and lengthy we write only the results in the following (see appendix):

$$V_{x0}^n\equiv ya_0^n(z)=y\omega\frac{\cos(k_xz)}{\cos(K_xd/2)} \tag{23}$$

$$\begin{aligned}
V_{x_1}^n\equiv A_\theta a_1^n(z)=&\frac{A_\theta\omega K_x^2}{\rho_\perp^n\cos(K_yd/2)(K_x^2-(K_y+\pi/d)^2)}\\
&\times\left\{\left[\rho_\perp^n\left(\frac{(\rho_\parallel^n-\rho_\perp^n)}{2\rho_\parallel^s(K_y+\pi/d)}+\frac{\rho_\parallel^nK_z^2K_y(K_y+\pi/d)+\rho(\rho_\perp^n-\rho_\parallel^n)K^2K_z^2}{2\rho K^2(K_z^2-(K_y+\pi/d)^2)(K_y+\pi/d)}\right)\right.\right.
\end{aligned}$$

$$-\frac{EK_y+B(K_y+\pi/d)}{\mathrm{i}\omega}\Bigg]$$

$$-\frac{(A+B+C+E)(K_y+\pi/d)K_z^2[\rho(\rho_\perp^n-\rho_\parallel^n)K^2+\rho_\parallel^s\rho_\parallel^n K_y(K_y+\pi/d)]}{\mathrm{i}\omega(K_z^2-(K_y+\pi/d)^2)\rho\rho_\parallel^n K^2}\Bigg\}$$

$$\times\left[\sin\left(K_y+\frac{\pi}{d}\right)z-\frac{\cos(K_yd/2)\sin(K_xz)}{\sin(K_xd/2)}\right]$$

$$+\frac{A_\theta\omega K_x^2}{\rho_\perp^n}\left(\frac{\tan(K_yd/2)\rho_\perp^n K_z[\rho(\rho_\perp^n-\rho_\parallel^n)K^2+\rho_\parallel^s\rho_\parallel^n K_y(K_y+\pi/d)]}{2\cos(K_zd/2)\rho\rho_\parallel^s(K_z^2-(K_y+\pi/d)^2)K^2}\right.$$

$$\left.-\frac{\tan(K_yd/2)K_z^3[\rho(\rho_\perp^n-\rho_\parallel^n)K^2+\rho_\parallel^s\rho_\parallel^n K_y(K_y+\pi/d)](A+B+C+E)}{\cos(K_zd/2)\rho\rho_\parallel^n K^2(K_z^2-(K_y+\pi/d)^2)\mathrm{i}\omega}\right)$$

$$\times\frac{1}{K_x^2-K_z^2}\left(\sin(K_zz)-\frac{\sin(K_zd/2)\sin(K_xz)}{\sin(K_xd/2)}\right)$$

$$+\frac{A_\theta\omega K_x^2(2K_x+\pi/d)B}{\rho_\perp^n\mathrm{i}\omega\cos(K_xd/2)(K_x^2-(K_x+\pi/d)^2)}$$

$$\times\left(\sin\left(K_x+\frac{\pi}{d}\right)z-\frac{\cos(K_xd/2)\sin(K_xz)}{\sin(K_xd/2)}\right)+\left(\frac{\pi}{d}\to-\frac{\pi}{d}\right)\tag{24}$$

$$V_{x2}^n\equiv A_\theta^2ya_2^n(z)=-\frac{A_\theta^2By\omega K_x}{32(A+B)\cos(K_xd/2)}\left[2z\sin(K_xz)-d\tan\left(\frac{K_xd}{2}\right)\cos(K_xz)\right.$$

$$\left.-\left(K_x+\frac{2\pi}{d}\right)\frac{\cos(K_x+2\pi/d)z+\cos(K_xz)}{(K_x+\pi/d)\pi/d}\right]$$

$$+\frac{1}{2}\left(1-\frac{\cos(K_xz)}{\cos(K_xd/2)}\right)A_Hy+\left(\frac{\pi}{d}\to-\frac{\pi}{d}\right)\tag{25}$$

$$V_{y_0}^n\equiv xb_0^n(z)=-\frac{x\omega\cos(K_yz)}{\cos(K_yd/2)}\tag{26}$$

$$V_{y_1}^n\equiv A_\theta xb_1^n(z)=0\tag{27}$$

$$V_{y_2}^n\equiv A_\theta^2xb_2^n(z)=\frac{A_\theta^2xK_y\omega}{2\rho_\perp^n}\left[\frac{(\rho_\parallel^s-\rho_\perp^s)(\rho_\perp^n-\rho_\parallel^n)}{4\rho_\parallel^s\cos(K_yd/2)}+\frac{\rho_\parallel^n-\rho_\perp^n}{4\cos(K_yd/2)}\right.$$

$$-\frac{K_z^2[\rho(\rho_\perp^n-\rho_\parallel^n)K^2+\rho_\parallel^s\rho_\parallel^n K_y(K_y+\pi/d)]}{2\rho\rho_\parallel^n\cos(K_yd/2)K^2}\left(-\frac{\rho_\parallel^n-\rho_\perp^n}{2(K_z^2-(K_y+\pi/d)^2)}\right.$$

$$\left.\left.+\frac{\rho_\parallel^n(\rho_\parallel^3-\rho_\perp^3)}{2\rho_\parallel^s(K_z^2-(K_y+\pi/d)^2)}+\frac{(2B+D+E)K_y(K_y+\pi/d)}{\mathrm{i}\omega(K_z^2-(K_y+\pi/d)^2)}\right)\right]$$

$$\times\left(z\sin(K_yz)-\frac{d}{2}\frac{\sin(K_yd/2)}{\cos(K_yd/2)}\cos(K_yz)\right)$$

$$+\frac{A_\theta^2xk_y^2\omega}{\rho_\perp^n}\left(\frac{k_Z^2[\rho(\rho_\perp^n-\rho_\parallel^n)k^2+\rho_\parallel^s\rho_\parallel^n Ky(Ky+\pi/d)]}{2\rho\rho_\parallel^n\cos(K_yd/2)K^2}\right)$$

$$\times\left(\frac{\sin(K_yd/2)\rho_\parallel^n(\rho_\parallel^s-\rho_\perp^s)}{\cos(K_zd/2)(K_z^2-(K_y+\pi/d)^2)\rho}-\frac{(\rho_\parallel^n-\rho_\perp^n)\sin(K_yd/2)}{\cos(K_zd/2)(K_z^2-(K_y+\pi/d)^2)}\right.$$

$$\left.+\frac{2(2B+D+E)\sin(K_yd/2)K_z(K_z+\pi/d)}{\cos(K_zd/2)\mathrm{i}\omega(K_z^2-(K_y+\pi/d)^2)}\right)\frac{1}{(K_y^2-(K_z+\pi/d)^2)}$$

$$\times\left[\cos\left(K_z+\frac{\pi}{d}\right)z+\frac{\sin(K_z d/2)}{\cos(K_y d/2)}\cos(K_y z)\right]$$

$$+\frac{A_\theta^2 x K_y^2\omega}{\rho_\perp^n}\left[\frac{(\rho_\parallel^s-\rho_\perp^s)(\rho_\perp^n-\rho_\parallel^n)}{2\rho_\parallel^s\cos(K_y d/2)}+\frac{\rho_\parallel^n-\rho_\perp^n}{4\cos(K_y d/2)}\right.$$

$$-\frac{K_z^2[\rho(\rho_\perp^n-\rho_\parallel^n)K^2+\rho_\parallel^s\rho_\parallel^n K_y(K_y+\pi/d)]}{2\rho\rho_\parallel^n\cos(K_y d/2)K^2}$$

$$\times\left(\frac{\rho_\parallel^n(\rho_\parallel^s-\rho_\perp^s)}{2\rho_\parallel^s(K_z^2-(K_y+\pi/d)^2}-\frac{\rho_\parallel^n-\rho_\perp^n}{2(K_z^2-(K_y+\pi/d)^2)}\right.$$

$$\left.\left.+\frac{(2B+D+E)(K_y+\pi/d)(K_y+2\pi/d)}{\mathrm{i}\omega(K_z^2-(K_y+\pi/d)^2)}\right)\right]\frac{1}{(4\pi/d)(K_y+\pi/d)}$$

$$\times\left[\cos\left(K_y+\frac{2\pi}{d}\right)z+\cos(K_y z)\right]+\frac{1}{2}\left(1-\frac{\cos(K_y z)}{\cos(K_y d/2)}\right)A_H x$$

$$+\left(\frac{\pi}{d}\to-\frac{\pi}{d}\right) \tag{28}$$

$$V_{z0}^n\equiv xC_0^n(z)=0 \tag{29}$$

$$V_{z1}^n\equiv A_\theta xC_1^n(z)=-A_\theta x\frac{\rho_\parallel^n\rho_\parallel^s\omega K_z^2K_y(K_y+\pi/d)+\rho(\rho_\perp^n-\rho_\parallel^n)\omega K^2K_z^2}{2\rho\rho_\parallel^n K^2\cos(K_y d/2)}$$

$$\times\frac{\cos(K_y+\pi/d)z\cos(K_z d/2)+\sin(K_y d/2)\cos(K_z z)}{(K_z^2-(K_y+\pi/d)^2)\cos(K_z d/2j)}+\left(\frac{\pi}{d}\to-\frac{\pi}{d}\right) \tag{30}$$

$$V_{z2}^n\equiv A_\theta^2 xC_2^n(z)=0 \tag{31}$$

$$V_{x0}^s\equiv a_0^s(z)=0 \tag{32}$$

$$V_{x1}^s\equiv A_\theta a_1^s(z)=-A_\theta\left[\left(\frac{(\rho_\perp^n-\rho_\parallel^n)\omega}{2\rho_\parallel^s(K_y+\pi/d)\cos(K_y d/2)}\right.\right.$$

$$\left.-\frac{\rho_\parallel^n\rho_\parallel^s\omega K_z^2K_y(K_y+\pi/d)+\rho(\rho_\perp^n-\rho_\parallel^n)\omega K^2K_z^2}{2\rho\rho_\parallel\cos(K_y d/2)(K_z^2-(K_y+\pi/d)^2)(K_y+\pi/d)K^2}\right)$$

$$\times\sin\left(K_y+\frac{\pi}{d}\right)z+\frac{\sin(K_y d/2)}{(K_z^2-(K_y+\pi/d)^2)\cos(K_z d/2)}$$

$$\left.\times\frac{\rho_\parallel^n\rho_\parallel^s\omega K_zK_y(K_y+\pi/d)+\rho(\rho_\perp^n-\rho_\parallel^n)\omega K^2K_z}{2K^2\rho\rho_\parallel^s\cos(K_y d/2)}\sin(K_z z)\right]$$

$$+\left(\frac{\pi}{d}\to-\frac{\pi}{d}\right) \tag{33}$$

$$V_{x2}^s\equiv A_\theta^2 a_2^s(z)+A_H y=A_H y \tag{34}$$

$$V_{z0}^s\equiv xa_0^s(z)=0 \tag{35}$$

$$V_{z1}^s\equiv A_\theta xC_1^s(z)=-A_\theta x\left(\frac{(\rho_\perp^n-\rho_\parallel^n)\omega}{\rho_\parallel^s\cos(K_y d/2)}\cos\left(\frac{\pi}{d}z\right)\cos(K_y z)+\frac{\rho_\parallel^n}{\rho_\parallel^s}V_{z1}^n\right) \tag{36}$$

$$V_{z2}^s\equiv A_\theta^2 xC_2^s(z)=0 \tag{37}$$

where the boundary conditions have been used in obtaining the above formulae, A_H is given in equation (A19) and

$$K_x^2\equiv\frac{\mathrm{i}\omega\rho_\perp^n}{2(A+B)}\equiv\frac{2\mathrm{i}}{\delta_x^2} \tag{38}$$

$$K_y^2 \equiv \frac{\mathrm{i}\omega\rho_\perp^n}{2(C+D+E)} \equiv \frac{2\mathrm{i}}{\delta_y^2} \tag{39}$$

$$K_z^2 \equiv \frac{\mathrm{i}\omega\rho_\parallel^n\rho}{2\rho_\parallel^s(2A+4B+C+D+2E)} \equiv \frac{2\mathrm{i}}{\delta_z^2} \tag{40}$$

$$K^2 \equiv \frac{\mathrm{i}\omega\rho_\parallel^n}{2(2B+D+E)} \equiv \frac{2\mathrm{i}}{\delta^2}. \tag{41}$$

The right-hand side of equations (38)–(41) have been defined on the basis of the fact that $A+B$, $C+D+E$, $2A+4B+C+D+2E$ and $2B+D+E$ are positive quantities. This is the case for temperatures smaller than $0.85T_c$ [3, 9, 13].

4. Calculation of the torque exerted on the slab

The torque exerted by the fluid on the oscillator is defined by

$$\Gamma_z = \frac{\mathrm{d}}{\mathrm{d}t}\int (xg_y - yg_x)\,\mathrm{d}V. \tag{42}$$

Furthermore we define $\Delta\Gamma_z \equiv \Gamma_z(\theta)-\Gamma_z(0)$, which is the variation in the exerted torque due to the appearance of the texture. In the uniform texture, hence $\Delta\Gamma_z = 0$. Straightforward calculations give

$$\begin{aligned}
\Delta\Gamma_z = A_\theta^2\pi\frac{a^4}{4}\frac{\mathrm{d}\omega}{\mathrm{d}t}\Bigg\{ &\tan\left(\frac{K_y d}{2}\right)\Bigg[\frac{(\rho_\perp^s-\rho_\parallel^s)(\rho_\perp^n-\rho_\parallel^n)\pi/d}{\rho_\parallel^s K_y(K_y+2\pi/d)} \\
&+\frac{(\rho_\parallel^s-\rho_\perp^s)[\rho(\rho_\perp^n-\rho_\parallel^n)K^2K_z^2+\rho_\parallel^s\rho_\parallel^n K_y(K_y+\pi/d)K_z^2]}{2\rho_\parallel^s\rho K^2} \\
&\times\left(\frac{2\pi/d}{(K_z^2-(K_y+\pi/d)^2)K_y(K_y+2\pi/d)}\right. \\
&\left.-\frac{2}{(K_z^2-(K_y+\pi/d)^2)(K_z+\pi/d)}\right)\Bigg] \\
&+\frac{1}{2}\rho_\perp^n\left(\frac{-d}{K_y\cos(K_yd/2)}+\frac{2\sin(K_yd/2)}{K_y^2}\right)\Bigg[\frac{K_y(\rho_\parallel^s-\rho_\perp^s)(\rho_\perp^n-\rho_\parallel^n)}{4\rho_\perp^n\rho_\parallel^s\cos(K_yd/2)} \\
&+\frac{K_y(\rho_\parallel^n-\rho_\perp^n)}{4\rho_\perp^n\cos(K_yd/2)}-\frac{K_yK_z^2[\rho(\rho_\perp^n-\rho_\parallel^n)K^2+\rho_\parallel^s\rho_\parallel^n K_y(K_y+\pi/d)]}{2\rho_\perp^n\rho\rho_\parallel^n\cos(K_yd/2)K^2} \\
&\times\left(-\frac{\rho_\parallel^n-\rho_\perp^n}{2(K_z^2-(K_y+\pi/d)^2)}+\frac{\rho_\parallel^n(\rho_\parallel^s-\rho_\perp^s)}{2\rho_\parallel^s(K_z^2-(K_y+\pi/d)^2)}\right. \\
&\left.+\frac{(2B+D+E)K_y(K_y+\pi/d)}{\mathrm{i}\omega(K_z^2-(K_y+\pi/d)^2)}\right)\Bigg] \\
&+\rho_\perp^n\left(\frac{2\cos(K_zd/2)}{K_z+\pi/d}+\frac{2\sin(K_zd/2)\sin(K_yd/2)}{\cos(K_yd/2)K_y}\right)\frac{K_y^2}{\rho_\perp^n(K_y^2-(K_z+\pi/d)^2)} \\
&\times\left(-\frac{K_z^2[\rho(\rho_\perp^n-\rho_\parallel^n)K^2+\rho_\parallel^s\rho_\parallel^n K_y(K_y+\pi/d)]}{2\rho\rho_\parallel^n\cos(K_yd/2)K^2}\right) \\
&\times\left(\frac{\sin(K_yd/2)\rho_\parallel^n(\rho_\parallel^s-\rho_\perp^s)}{\rho\cos(K_zd/2)(K_z^2-(K_y+\pi/d)^2)}-\frac{(\rho_\parallel^n-\rho_\perp^n)\sin(K_yd/2)}{\cos(K_zd/2)(K_z^2-(K_y+\pi/d)^2)}\right.
\end{aligned}$$

$$+\frac{2(2B+D+E)\sin(K_y d/2)K_z(K_z+\pi/d)}{\cos(K_z d/2)\mathrm{i}\omega(K_z^2-(K_y+\pi/d)^2)}\Bigg)$$

$$+\frac{K_y^2}{4}\Bigg[\frac{(\rho_\parallel^s-\rho_\perp^s)(\rho_\perp^n-\rho_\parallel^n)}{2\rho_\parallel^s\cos(K_y d/2)}+\frac{\rho_\parallel^n-\rho_\perp^n}{4\cos(K_y d/2)}$$

$$-\frac{K_z^2[\rho(\rho_\perp^n-\rho_\parallel^n)K^2+\rho_\parallel^s\rho_\parallel^n K_y(K_y+\pi/d)}{2\rho\rho_\parallel^n\cos(K_y d/2)K^2}\Bigg(\frac{\rho_\parallel^n(\rho_\parallel^s-\rho_\perp^s)}{2\rho_\parallel^s(K_z^2-(K_y+\pi/d)^2)}$$

$$-\frac{\rho_\parallel^n-\rho_\perp^n}{2(K_z^2-(K_y+\pi/d)^2)}+\frac{(2B+D+E)(K_y+\pi/d)(K_y+2\pi/d)}{\mathrm{i}\omega(K_z^2-(K_y+\pi/d)^2)}\Bigg)\Bigg]$$

$$\times\frac{1}{(4\pi/d)(K_y+\pi/d)}\frac{2\sin(K_y d/2)(2\pi/d)}{K_y(K_y+2\pi/d)}-(\rho_\parallel^n-\rho_\perp^n)\frac{\tan(K_y d/2)}{2}$$

$$\times\frac{2\pi/d}{K_y(K_y+2\pi/d)}+(\rho_\parallel^n-\rho_\perp^n)\frac{-K_z^2[\rho(\rho_\perp^n-\rho_\parallel^n)K^2+\rho_\parallel^s\rho_\parallel^n K_y(K_y+\pi/d)]}{2\rho\rho_\parallel^n\cos(K_y d/2)K^2}$$

$$\times\Bigg(\frac{\sin(K_y d/2)(2\pi/d)}{(K_z^2-(K_y+\pi/d)^2)(K_y+2\pi/d)K_y}$$

$$+\frac{2\sin(K_y d/2)}{(K_z^2-(K_y+\pi/d)^2)(K_z+\pi/d)}\Bigg)$$

$$+\rho_\perp^n\frac{BK_x}{16(A+B)\cos(K_x d/2)}\left(\frac{-dK_x+\sin(K_x d)}{K_x^2\cos(K_x d/2)}-\frac{2\sin(K_x d/2)}{(K_x+\pi/d)K_x}\right)\Bigg\}$$

$$+\pi\frac{a^4}{4}\rho_\perp^n\left(\frac{\tan(K_x d/2)}{K_x}-\frac{\tan(K_y d/2)}{K_y}\right)\frac{\mathrm{d}A_H}{\mathrm{d}t}+\left(\frac{\pi}{d}\to-\frac{\pi}{d}\right) \tag{43}$$

where a is the radius of the oscillating slab. We write $\Delta\Gamma_z$ in terms of the dimensionless quantities ΔF_1 and ΔF_2 as

$$\Delta\Gamma_z=\pi\rho_\perp^n\frac{\mathrm{d}\omega}{\mathrm{d}t}a^4d(\Delta F_1+\mathrm{i}\Delta F_2) \tag{44}$$

where F_1 and F_2 are related to the inertial and dissipative effects, respectively. Here we calculate ΔF_1 and ΔF_2 for the case in which K_xd, K_yd, K_zd and Kd are much smaller than one; hence

$$\Delta\Gamma_z=\pi\rho_\perp^n\frac{\mathrm{d}\omega}{\mathrm{d}t}a^4d\frac{A_\theta^2}{8}\frac{\rho_\perp^s(-\rho_\parallel^n+\rho_\perp^n)}{\rho_\parallel^s\rho_\perp^n}+\mathrm{O}(\mathrm{d}K_i) \tag{45}$$

where K_i stands for K_x, K_y, K_z and K. As is obvious, $\Delta F_2=0$ for this case.

5. Concluding remarks

The hydrodynamic equations for a narrow slab when the state of ^{3}He-A is deformed slightly by a perpendicular magnetic field are solved for planar textures. The components of the normal and superfluid velocity are obtained in equations (23)–(37). $\Delta\Gamma_z$ or the values of ΔF_1 and ΔF_2 are proportional to A_θ^2 which are plotted versus h in figure 1. ΔF_1 is proportional to the change in the resonant frequency and ΔF_2 to its width. As we have mentioned previously, our calculations are exact for low temperatures $t\equiv T/T_c<0.8$ and $d<d_c$. For the special case in which K_xd, K_yd, K_zd and Kd are much smaller than one, $\Delta F_2=0$ and ΔF_1 is given by

$$\Delta F_1=\frac{A_\theta^2}{8}\frac{\rho_\perp^s(\rho_\perp^n-\rho_\parallel^n)}{\rho_\parallel^s\rho_\perp^n}. \tag{46}$$

One may obtain the values of some of the components of ρ^s and ρ^n by comparing equation (46) with the experimental results. We mention that the dependence of A_θ^2 on the parameters H_D, H_D, K_S and K_S can be determined theoretically, since the texture is insensitive to these values [4].

Acknowledgments

I am grateful to Professor H E Hall and Dr J R Hook for useful discussions on this work.

Appendix A. Calculation of the components of the velocities

In this appendix we derive briefly the velocity components which are written in equations (23)–(37). By substituting equation (20) into equation (19) we have

$$-\mathrm{i}\omega\rho V_z^s + 2\frac{\partial}{\partial z}\left(B\sin(2\theta) + \frac{D}{2}\sin(2\theta)\cos^2\theta + \frac{E}{2}\sin(2\theta)\right)\frac{\partial V_y^n}{\partial z}$$
$$+2\frac{\partial}{\partial z}(2A + 4B\cos^2\theta + C + D\cos^4\theta + 2E\cos^2\theta)\frac{\partial V_z^n}{\partial z} = 0. \quad \text{(A1)}$$

The zeroth-order terms of equation (20) give

$$\rho_\parallel^s V_{z0}^s = -\rho_\parallel^n V_{z0}^n. \quad \text{(A2)}$$

The zeroth-order terms of equation (A1) give

$$K_z^2 V_{z0}^n + \frac{\partial^2 V_{z0}^n}{\partial z^2} = 0. \quad \text{(A3)}$$

The solution of the above equation with the boundary condition $V^n = r\omega\hat{\phi}$ at $z = \pm d/2$ is $V_{z0}^n = 0$; hence from (A2) we get $V_{z0}^s = 0$. These results are written in equations (29) and (35). The zeroth-order terms of equation (18) give

$$\frac{\rho_\perp^s - \rho}{\rho_\perp^n} K_y^2 V_{y_0}^s + K_y^2 V_{y_0}^n + \frac{\partial^2 V_{y_0}^n}{\partial z^2} = 0. \quad \text{(A4)}$$

The solution of equation (A4) with the boundary conditions and Mermin–Ho relation, equation (14), which gives $V_{y_0}^s \neq y_{y_0}^s(z)$, is

$$V_{y_0}^n = \left(-\frac{x\omega}{\cos(K_y d/2)}\right)\cos(K_y z) + \frac{\rho_\perp^s - \rho}{\rho_\perp^n}\left(1 - \frac{\cos(K_y z)}{\cos(K_y d/2)}\right)V_{y_0}^s. \quad \text{(A5)}$$

The form of the velocity field in equation (16) and the second equation in (21) lead us to write $V_{y_0}^s = \text{constant}\, x$. The boundary conditions on V^n, g_r and g_z give $V_{y_0}^s = V_{x_0}^s = 0$. The first-order terms of equations (19) and (20) give

$$\frac{\partial^2 V_{z1}^n}{\partial z^2} + K_z^2 V_{z1}^n = \frac{-A_\theta x\omega\rho_\parallel^s K_z^2}{2\rho\rho_\parallel^n K^2\cos(K_y d/2)}\left[\frac{\rho(\rho_\perp^n - \rho_\parallel^n)}{\rho_\parallel^s}K^2 + \rho_\parallel^n\left(K_y + \frac{\pi}{d}\right)K_y\right]$$
$$\times\cos\left(K_y + \frac{\pi}{d}\right)z \quad \text{(A6)}$$

and

$$V_{z1}^s = -\frac{A_\theta x\omega(\rho_\parallel^n - \rho_\perp^n)\cos(\pi/d)z\cos(K_y z)}{\rho_\parallel^s\cos(K_y d/2)} - \frac{\rho_\parallel^n}{\rho_\parallel^3}V_{z1}^n. \quad \text{(A7)}$$

The solution of equations (A6) and (A7) are written in equations (30) and (36). From the Mermin–Ho relation, $\partial V_{z1}^s/\partial x = \partial V_{x1}^s/\partial z$ and the boundary conditions, we obtained V_{x1}^s

which is written in equation (33). It is clear that an average over z of V^s_{x1} is zero. This means that there is no mass flow in the x direction to the first-order contribution.

The first-order terms in equation (18) give

$$\frac{\rho^s_\perp - \rho}{\rho^n_\perp} K_y^2 V^s_{y1} + K_y^2 V^n_{y1} + \frac{\partial^2 V^n_{y1}}{\partial z^2} = 0. \tag{A8}$$

The solution of equation (A8) is similar to equations (A4) and (A5) and we obtain $V^n_{y1} = V^s_{y1} = 0$; these are written in equations (27) and the last equation of (21).

The second-order terms in equation (20) give

$$\rho^s_\parallel V^s_{z2} + \rho^n_\parallel V^n_{2z} = 0. \tag{A9}$$

The second-order terms in equation (19) and equation (A9) give

$$K^2_{z2} V^n_{z2} + \frac{\partial^2 V^n_{z2}}{\partial z^2} = 0 \tag{A10}$$

where

$$K^2_{z2} = \frac{\mathrm{i}\omega\rho\rho^n_\parallel}{2(2A+4B+C+D+2E)\rho^s_\parallel}.$$

The solution of equation (A10) with the boundary condition on $\boldsymbol{V}^n$ is zero; hence $V^n_{z2} = V^s_{z2} = 0$.

The first-order terms in equation (19) give

$$\frac{\partial^2 V^n_{z1}}{\partial z^2} + K_z^2 V^n_{12} = -\frac{A_\theta x\omega K_z^2 \rho^s_\parallel}{2\rho\rho^n_\parallel K^2 \cos(K_y d/2)} \left[\frac{\rho(\rho^n_\perp - \rho^n_\parallel)K^2}{\rho^s_\parallel} + \rho^n_\parallel \left(K_y + \frac{\pi}{d} \right) K_y \right] \times \cos\left(K_y + \frac{\pi}{d} \right) z + \left(\frac{\pi}{d} \to -\frac{\pi}{d} \right). \tag{A11}$$

The solution of equation (A11) is written in equation (30).

The zero-, first- and second-order terms of equation (17) give

$$\frac{\partial^2 V^n_{x0}}{\partial z^2} + K_x^2 V^n_{x0} = 0 \tag{A12}$$

$$\begin{aligned} \frac{\partial^2 V^n_{x1}}{\partial z^2} + K_x^2 V^n_{x1} = & -\frac{\mathrm{i}\omega(\rho^s_\perp - \rho)}{2(A+B)} V^s_{x1} - \frac{A_\theta E\omega K_y \cos[(\pi/d)z]\sin(K_y z)}{(A+B)\cos(K_y d/2)} \\ & + \frac{A_\theta B K_x \omega \cos[(\pi/d)z]\sin(K_x z)}{(A+B)\cos(K_x d/2)} \\ & - \frac{A_\theta B\omega}{A+B}\frac{\partial}{\partial z}\left[\cos\left(\frac{\pi}{d}z\right)\left(-\frac{\cos(K_y z)}{\cos(K_y d/2)} + \frac{\cos(K_x z)}{\cos(K_x d/2)} \right) \right] \\ & - \frac{A+B+C+E}{A+B}\frac{1}{x}\frac{\partial V^n_{z1}}{\partial z} \end{aligned} \tag{A13}$$

and

$$\frac{\partial^2 V^n_{x2}}{\partial z^2} + K_x^2 V^n_{x2} = B\frac{K_x^2}{\mathrm{i}\omega\rho^n_\perp}\frac{\partial}{\partial z}\theta^2 \frac{\partial V^n_{x0}}{\partial z} + K_x^2 V^s_{x2}. \tag{A14}$$

The solutions of equations (A12)–(A14) are written in equations (23)–(25). It is clear that an average on z of V^n_{x1} is zero. Hence, there is no mass flow totally in the x direction to the first-order contribution.

The zero- and second-order terms of equation (18) give

$$\frac{\partial^2 V^n_{y0}}{\partial z^2} + K_y^2 V^n_{y0} = 0 \tag{A15}$$

and

$$\frac{\partial^2 V^n_{y2}}{\partial z^2} + K_y^2 V^n_{y2} = \frac{K_y^2}{\mathrm{i}\omega\rho^n_\perp}\bigg(-\mathrm{i}\omega(\rho^s_\parallel - \rho^s_\perp)\theta V^s_z - \mathrm{i}\omega(\rho^n_\parallel - \rho^n_\perp)\theta V^n_{z1} - \mathrm{i}\omega(\rho^n_\parallel - \rho^n_\perp)\theta^2 V^n_{y0}$$
$$-2(2B+D+E)\frac{\partial}{\partial z}\theta\frac{\partial V^n_{z1}}{\partial z}\bigg) + K_y^2 V^s_{y2}. \tag{A16}$$

The solutions of equations (A15) and (A16) are written in equations (26) and (28). Finally by using the boundary condition $\int_{-d/2}^{d/2} g_r \, \mathrm{d}z = \mathbf{0}$, the Mermin–Ho relation and the form of $\boldsymbol{V}^s$ in equation (16), we get

$$V^s_{y2} = A_H x \tag{A17}$$
$$V^s_{x2} = A_H y \tag{A18}$$

where

$$A_H = \left[2(\rho^n_\perp + \rho^s_\perp) + \frac{2\rho^n_\perp}{K_z d}\tan\left(\frac{K_z d}{2}\right) - \frac{2\rho^n_\perp}{K_y d}\tan\left(\frac{K_y d}{2}\right)\right]^{-1}\omega A_\theta^2$$
$$\times\Bigg\{ -\frac{B\rho^n_\perp}{16(A+B)}\left[1+\tan^2\left(\frac{K_x d}{2}\right) - \frac{2\pi/d}{K_x(K_x d+\pi)}\tan\left(\frac{K_x d}{2}\right)\right]$$
$$-\Bigg[\frac{(\rho^s_\parallel - \rho^s_\perp)(\rho^n_\perp - \rho^n_\parallel)}{4\rho^s_\parallel} + \tfrac{1}{4}(\rho^n_\parallel - \rho^n_\perp)$$
$$-\frac{K_z^2[\rho(\rho^n_\perp - \rho^n_\parallel)K^2 + \rho^s_\parallel\rho^n_\parallel K_y(K_y+\pi/d)]}{2\rho\rho^n_\parallel K^2}\bigg(-\frac{\rho^n_\parallel - \rho^n_\perp}{2(K_z^2 - (K_y+\pi/d)^2)}$$
$$+\frac{\rho^n_\parallel(\rho^s_\parallel - \rho^s_\perp)}{2\rho^s_\parallel(K_z^2 - (K_y+\pi/d)^2)} + \frac{(2B+D+E)K_y(K_y+\pi/d)}{\mathrm{i}\omega(K_z^2 - (K_y+\pi/d)^2)}\bigg)\Bigg]$$
$$\times\left[-1 + \frac{2}{K_y d}\tan\left(\frac{K_y d}{2}\right) - \tan^2\left(\frac{K_y d}{2}\right)\right]$$
$$+\frac{K_z^2[\rho(\rho^n_\perp - \rho^n_\parallel)K^2 + \rho^s_\parallel\rho^n_\parallel K_y(K_y+\pi/d)]\tan(K_y d/2)}{2\rho\rho^n_\parallel K^2(K_z^2 - (K_y+\pi/d)^2)}$$
$$\times\left(\frac{\rho^n_\parallel(\rho^s_\parallel - \rho^s_\perp)}{\rho} - (\rho^n_\parallel - \rho^n_\perp) + \frac{2(2B+D+E)K_z(K_z+\pi/d)}{\mathrm{i}\omega}\right)$$
$$\times\frac{2K_y^2}{(K_y^2 - (K_z+\pi/d)^2)}\left(-\frac{1}{K_z d+\pi} + \frac{\tan(K_z d/2)\tan(K_y d/2)}{K_y d}\right)$$
$$+\Bigg[\frac{(\rho^s_\parallel - \rho^s_\perp)(\rho^n_\perp - \rho^n_\parallel)}{2\rho^s_\parallel} + \left(\frac{\rho^n_\parallel - \rho^n_\perp}{4}\right)$$
$$-\frac{K_z^2[\rho(\rho^n_\perp - \rho^n_\parallel)K^2 + \rho^s_\parallel\rho^n_\parallel K_y(K_y+\pi/d)]}{2\rho\rho^n_\parallel\cos(K_y d/2)K^2}\Bigg]\frac{K_y^2}{(K_z^2 - (K_y+\pi/d)^2)}$$
$$\times\left(\frac{\rho^n_\parallel(\rho^s_\parallel - \rho^s_\perp)}{2\rho^s_\parallel} - \frac{\rho^n_\parallel - \rho^n_\perp}{2} + \frac{(2B+D+E)(K_y+\pi/d)(K_y+2\pi/d)}{\mathrm{i}\omega(K_z^2 - (K_y+\pi/d)^2)}\right)$$
$$\times\frac{d^2\tan(K_y d/2)}{\rho^n_\perp\pi d K_y(K_y d+2\pi)} + \frac{2\pi^2\rho^s_\perp(\rho^s_\perp - \rho^s_\parallel)\tan(K_y d/2)}{\rho^s_\parallel K_y d(4\pi^2 - K_y^2 d^2)}$$
$$+\frac{2\pi(\rho^s_\perp - \rho^s_\parallel)[\rho^n_\parallel\rho^s_\parallel K_z^2 K_y(K_y+\pi/d) + \rho(\rho^n_\perp - \rho^n_\parallel)K^2K_z^2]\tan(K_y d/2)}{\rho^s_\parallel\rho^n_\parallel d K^2(K_z^2 - (K_y+\pi/d)^2)(K_y d+2\pi)(K_z^2 d^2 - \pi^2)K_y}\Bigg\}. \tag{A19}$$

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